American Journal of Corporate Management, Vol.2, No.2 (February, 2025),
E-ISSN: 2997-9404

American Journal of Corporate Management
https://semantjournals.org/index.php/ AJCM

| Research Article

The concept of derivative

Oramov Jura
University of Economics and Pedagogy, Associate Professor of the "Mathematics" Department

Abstract: Derivatives are often a person's introduction into the world of calculus. And it makes sense. Derivatives are really just an extension of the concepts learned in algebra, trigonometry, etc. However, I think the finer points of derivatives are often overlooked.

Keywords: derivative, math, algebra, calculus, concept.

Introduction

The concept of a derivative is a fundamental principle in calculus and plays a crucial role in various fields such as mathematics, physics, economics, and engineering. A derivative represents the rate of change of a function with respect to its independent variable, providing a mathematical framework for understanding dynamic systems and optimization problems. The derivative is primarily defined as the limit of the difference quotient, which measures how a function's output changes as its input varies. This concept is essential in analyzing instantaneous rates of change, such as velocity in physics, marginal cost in economics, and sensitivity analysis in finance. Moreover, derivatives are widely used in solving optimization problems, modeling growth and decay processes, and predicting trends in real-world applications. This study aims to explore the definition, properties, and applications of derivatives, highlighting their significance in both theoretical and practical contexts. By examining the fundamental principles of differentiation and its role in mathematical modeling, this research provides a deeper understanding of how derivatives contribute to various scientific and technological advancements. There are two common ways of notating a derivative: Newton notation and Leibniz notation.

Methodology

The notations are named after Issac Newton and Gottfried Wilhelm Leibniz, respectively. In Newton notation, the derivative of f(x) is denoted as

$$\text{Derivative of } f(x)\text{: } f'(x)$$

and in Leibniz notation, the derivative of f(x) is denoted as

$$\text{Derivative of } f(x)\text{: } \frac{df}{dx}$$

There are benefits and downsides to both notations. Newton's notation is simpler whereas Leibniz notation provides the "units" or variable with which we are differentiating. Leibniz notation is especially helpful in engineering or scientific applications since units (time, distance, etc.) are especially important.

Both notations are often used interchangeably. It is more of a personal preference regarding which notation you use. There are benefits and downsides to both notations. Newton's notation is simpler whereas Leibniz notation provides the "units" or variable with which we are differentiating. Leibniz notation is especially helpful in engineering or scientific applications since units (time, distance, etc.) are especially important. Both notations are often used interchangeably. It is more of a personal preference regarding which notation you use. Okay, let's get started! We know how to notate a derivative, but what is it? A derivative is an instantaneous rate of change with respect to one or more variables of a function. Let's examine each part of this definition

Results and discussion

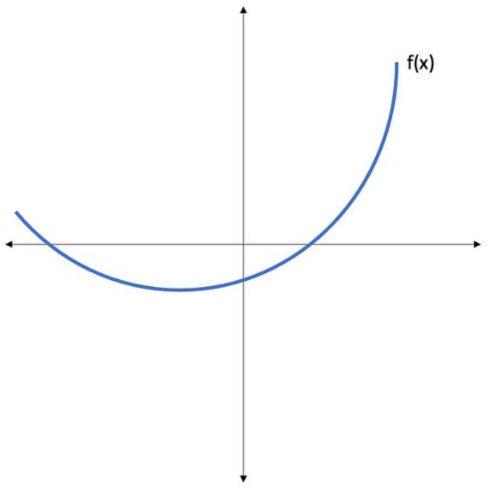
"…instantaneous…": when we determine the derivative, we are evaluating the change at a specified point rather than the average rate of change over the entire function.

"…rate of change…": we have encountered rate of change in algebra when we determined the slope of a given line.

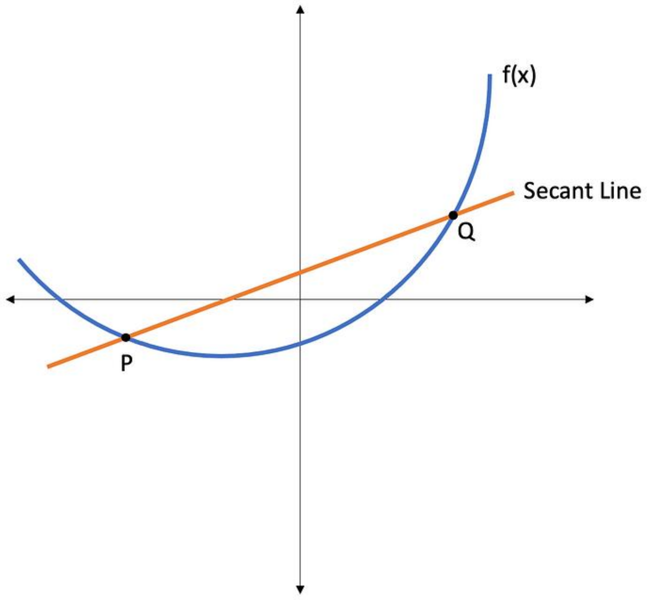
"…with respect to one or more variables…": in single variable calculus, we analyze functions of one variable (e.g. f(x)), but a derivative is not limited to single variable functions. We can take derivatives of functions with N number of variables (e.g. f(x, y, z)).

Simply put, we are finding the slope of the tangent line at a particular point. It measures the variation of slope as the point progresses along the curve.

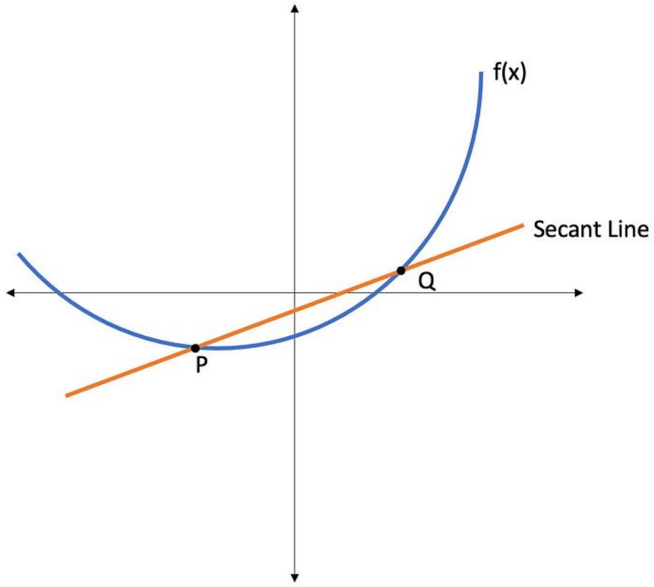
Geometric and Algebraic Interpretation



Let’s start with a simple curve produced by the function f(x). The actual function itself doesn’t matter, but let’s assume it produces the curve above.



Next, let’s create an arbitrary secant line that crosses at points P and Q. We will denote this as PQ. What happens as we bring points P and Q closer and closer together?



Notice that the distance between P and Q shortened. What if we kept bringing P and Q closer together? What happens to the distance between P and Q? Eventually, P and Q would become one point! However, a line that touches a function at one point is a tangent line. Therefore, we can say that the slope of the secant line as the distance between P and Q approaches zero is the definition of a derivative at that point! Let's see this in algebraic form using limits.

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Conclusion

The above expression looks a bit different and it is missing P and Q. However, notice that it still represents what we have already discovered. Let's look at the numerator first. This is simply a rewriting of the values at Q and P, respectively. P is simply the function evaluated at x and Q is the function evaluated at x plus some small change in x. The denominator is the change in x from P to Q. Then, decrease the distance between P and Q (or take the limit) as the change in x approaches 0. Notice that the definition of a derivative is simply the traditional "rise over run" definition of the slope. It is algebra disguised at calculus. The only difference is that we are taking the distance between two points of a secant line, shrinking the distance at infinity, and then finding the slope of the now tangent line. Both notations are often used interchangeably. It is more of a personal preference regarding which notation you use. There are benefits and downsides to both notations. Newton's notation is simpler whereas Leibniz notation provides the "units" or variable with which we are differentiating. Leibniz notation is especially helpful in engineering or scientific applications since units (time, distance, etc.) are especially important. Both notations are often used interchangeably. It is more of a personal preference regarding which notation you use. Okay, let's get started! We know how to notate a derivative, but what is it? A derivative is an instantaneous rate of change with respect to one or more variables of a function. Let's examine each part of this definition

References

1. Rakhmatullaev, A. I., Ruziev, Z. I., & Normamatov, I. B. (2020). Advantages of Developing Cashless Settlement. Asian Journal of Technology & Management Research [ISSN: 2249–
201

0892], 10(01).

2. Ikromovich, R. Z., & G'ayratovich, A. O. (2023). PROBLEMS OF LIBRARY AND INCREASE COMPETITIVENESS OF NON-GOVERNMENT EDUCATIONAL INSTITUTIONS IN OUR COUNTRY. Open Access Repository, 4(3), 201-206.
3. Alimkhanova, N. (2023). Improving accounting and auditing of goods and material resources. Экономика и социум, (4-2 (107)), 21-25.
4. Oramov, J. (2023). INTERNATIONAL CORPORATIONS AS SUBJECTS OF INTERNATIONAL FINANCIAL RELATIONS. Экономика и социум, (4-2 (107)), 234-238.
5. Nigora, A. (2023). Improving Accounting And Auditing Of Goods And Material Resources. Gospodarka i Innowacje., 208-215.
6. Alimkhanova, N. (2023). Improving accounting and auditing of goods and material resources. Экономика и социум, (4-2 (107)), 21-25.
7. Murodova, N. U., Temirova, F. S., Alimkhanova, N. A., Dostova, M. K., & Azimova, K. E. IMPROVING THE CALCULATION OF INVENTORIES OF GOODS IN ACCOUNTING. Kielce: Laboratorium Wiedzy Artur Borcuch.
8. Ikramovich, R. Z., & Khamidovich, T. S. (2022). The Role of Fiscal Policy in Ensuring the Financial Stability of Uzbekistan. European Multidisciplinary Journal of Modern Science, 4, 361-366.
9. Ikramovich, R. Z., & Khamidovich, T. S. (2022). The Role of Fiscal Policy in Ensuring the Financial Stability of Uzbekistan. European Multidisciplinary Journal of Modern Science, 4, 361-366.
10. Jakhongir, O. (2023). Digital Economy of the Age.
11. Ikromovich, R. Z., Murtazaevna, K. Y., Rashidovich, M. N., & Oramov, J. J. FEATURES OF THE CREDIT MECHANISM OF INTERNATIONAL FINANCIAL ORGANIZATIONS.
12. Nigora, A. (2022). Monetary policy. Web of Scientist: International Scientific Research Journal, 3(10), 585-591.