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| Research Article

Mathematical induction method and its applications

Oramov Jura
University of Economics and Pedagogy, Associate Professor of the "Mathematics" Department

Abstract: Mathematical Induction is a technique of proving a statement, theorem or formula which is thought to be true, for each and every natural number. Mathematical Induction is a powerful and elegant technique for proving certain types of mathematical statements: general propositions which assert that something is true for all positive integers or for all positive integers from some point on.

Keywords: Mathematical Induction, technique, statement, theory, formula.

A class of integers is called hereditary if, whenever any integer x belongs to the class, the successor of x (that is, the integer x + 1) also belongs to the class. The principle of mathematical induction is then: If the integer 0 belongs to the class F and F is hereditary, every nonnegative integer belongs to F. Alternatively, if the integer 1 belongs to the class F and F is hereditary, then every positive integer belongs to F. The principle is stated sometimes in one form, sometimes in the other. As either form of the principle is easily proved as a consequence of the other, it is not necessary to distinguish between the two.

The principle is also often stated in intensional form: A property of integers is called hereditary if, whenever any integer x has the property, its successor has the property. If the integer 1 has a certain property and this property is hereditary, every positive integer has the property.

Proof by mathematical induction

An example of the application of mathematical induction in the simplest case is the proof that the sum of the first n odd positive integers is n2—that is, that

(1.) $1 + 3 + 5 +\cdots+ (2n - 1) = n2$

for every positive integer n. Let F be the class of integers for which equation (1.) holds; then the integer 1 belongs to F, since 1 = 12. If any integer x belongs to F, then

(2.) $1 + 3 + 5 +\cdots+ (2x - 1) = x2.$

The next odd integer after 2x − 1 is 2x + 1, and, when this is added to both sides of equation (2.), the result is

(3.) 1 + 3 + 5 +⋯+ (2x + 1) = x2 + 2x + 1 = (x + 1)2.

Equation (2.) is called the hypothesis of induction and states that equation (1.) holds when n is x, while equation (3.) states that equation (1.) holds when n is x + 1. Since equation (3.) has been proved as a consequence of equation (2.), it has been proved that whenever x belongs to F the successor of x belongs to F. Hence by the principle of mathematical induction all positive integers belong to F.

Henri Poincaré maintained that mathematical induction is synthetic and a priori—that is, it is not reducible to a principle of logic or demonstrable on logical grounds alone and yet is known independently of experience or observation. Thus mathematical induction has a special place as constituting mathematical reasoning par excellence and permits mathematics to proceed from its premises to genuinely new results, something that supposedly is not possible by logic alone. In this doctrine Poincaré has been followed by the school of mathematical intuitionism which treats mathematical induction as an ultimate foundation of mathematical thought, irreducible to anything prior to it and synthetic a priori in the sense of Immanuel Kant.

In mathematics, the role of induction is largely that it underlies the chosen axiomatics. After long-term practice showed that the straight path is always shorter than the crooked or broken one, it was natural to formulate the axiom: for any three points A, B and C, the inequality AB + BC □ AC holds.

Complete and incomplete induction

Inductive inference is a form of abstract thinking in which thought develops from knowledge of a lower degree of generality to knowledge of a higher degree of generality, and the conclusion following from the premises is primarily probabilistic in nature.

Taking into account the dependence on the nature of the study, a distinction is made between complete and incomplete induction.

Complete induction is an inference in which a general conclusion is made on the basis of studying all objects or phenomena of a given class. In this case, the reasoning has the following pattern:

For example, establishing that each of the documents necessary to assess the readiness of a criminal case to be transferred to court is available allows us to conclude with full justification that the case should be transferred to court Complete induction provides reliable knowledge, since the conclusion is made only about those objects or phenomena that are listed in the premises. But the scope of application of complete induction is very limited.

Complete induction can be applied when it becomes possible to deal with a closed class of objects, the number of elements in which is finite and easily observable. It presupposes the presence of the following conditions:

a) exact knowledge of the number of objects or phenomena to be

studied;

b) conviction that the feature belongs to each element of the class;

c) a small number of elements of the class being studied;

d) expediency and rationality.

The method of mathematical induction is widely used in proving theorems, identities, inequalities, in solving divisibility problems, in solving some geometric and many other problems.

The most natural application of the method of mathematical induction in geometry, close to the use of this method in number theory and in algebra, is its application to solving geometric problems on computation.

The method of mathematical induction is an important way of proving propositions (statements) that depend on a natural argument.

The method of mathematical induction is as follows:

A proposition (statement) P(n), depending on a natural number n, is true for any natural n if:

1. P(1) is a true proposition (statement);

2. P(n) remains a true proposition (statement) if n is increased by one, that is, P(n + 1) is a true proposition (statement).

Thus, the method of mathematical induction involves two stages:

1. Verification stage: it is checked whether the proposition (statement) P(1) is true.

2. Proof stage: it is assumed that the proposition P(n)

is true, and the truth of the proposition P(n + 1) is proved (n is increased by one).

Remark 1. In some cases, the method of mathematical induction is used in the following form:

Let m be a natural number, $m > 1$, and P(n) be a proposition depending on n, $n \geq m$.

If

1. P(m) is true;

2. P(n), being a true proposition, implies the truth

of the proposition P(n + 1) for any natural n, $n \geq m$, then P(n) is a

true proposition for any natural n, $n \geq m$.

Below, we will consider examples of the application of the method

of mathematical induction.

Let us consider several examples.

Example 1. Into how many triangles can an n-gon (not necessarily convex) be divided by its non-intersecting diagonals?

Solution.

For a triangle, this number is equal to one (not a single diagonal can be drawn in a triangle); for a quadrilateral, this number is, obviously, equal to two.

References

1. Nigora, A. (2023). Improving Accounting And Auditing Of Goods And Material Resources. Gospodarka i Innowacje., 208-215.
2. Alimkhanova, N. (2023). Improving accounting and auditing of goods and material resources. Экономика и социум, (4-2 (107)), 21-25.
3. Murodova, N. U., Temirova, F. S., Alimkhanova, N. A., Dostova, M. K., & Azimova, K. E. IMPROVING THE CALCULATION OF INVENTORIES OF GOODS IN ACCOUNTING. Kielce: Laboratorium Wiedzy Artur Borcuch.
4. Alimkhanova, N. (2023). Improving accounting and auditing of goods and material resources. Экономика и социум, (4-2 (107)), 21-25.
5. Nigora, A. (2022). Monetary policy. Web of Scientist: International Scientific Research Journal, 3(10), 585-591.

6. Rakhmatullaev, A. I., Ruziev, Z. I., & Normamatov, I. B. (2020). Advantages of Developing Cashless Settlement. Asian Journal of Technology & Management Research [ISSN: 2249–0892], 10(01).
7. Ikromovich, R. Z., & G'ayratovich, A. O. (2023). PROBLEMS OF LIBRARY AND INCREASE COMPETITIVENESS OF NON-GOVERNMENT EDUCATIONAL INSTITUTIONS IN OUR COUNTRY. Open Access Repository, 4(3), 201-206.
8. Ikramovich, R. Z., & Khamidovich, T. S. (2022). The Role of Fiscal Policy in Ensuring the Financial Stability of Uzbekistan. European Multidisciplinary Journal of Modern Science, 4, 361-366.
9. Ikramovich, R. Z., & Khamidovich, T. S. (2022). The Role of Fiscal Policy in Ensuring the Financial Stability of Uzbekistan. European Multidisciplinary Journal of Modern Science, 4, 361-366.
10. Jakhongir, O. (2023). Digital Economy of the Age.
11. Oramov, J. (2023). INTERNATIONAL CORPORATIONS AS SUBJECTS OF INTERNATIONAL FINANCIAL RELATIONS. Экономика и социум, (4-2 (107)), 234-238.
12. Ikromovich, R. Z., Murtazaevna, K. Y., Rashidovich, M. N., & Oramov, J. J. FEATURES OF THE CREDIT MECHANISM OF INTERNATIONAL FINANCIAL ORGANIZATIONS.