

Teaching Logarithmic Equations to Academic Lyceum Students on the Base of New Pedagogical Technologies

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Abstract: This article contains ideas about solving problems related to teaching the subject of logarithmic equations to students of an academic lyceum, and easy ways to deliver the subject well to students. The lesson shows how to clearly convey the topic to students and several ways to solve equations. Various methods and methods were used during the passage of the subject in order to find out how the students mastered the subject. This article uses "Mosini Top", "Mathematical Lotto" and "Rocket" methods.

Keywords: logarithm, root, external root, graph, coordinate, logarithmization, same base, lotto, rocket, method.



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Introduction

This article focuses on the effective teaching methods for solving logarithmic equations to students at an academic lyceum. The primary goal is to present easy and clear ways to teach this challenging topic, ensuring that students grasp the fundamental concepts. It outlines various teaching techniques, including the "Mosini Top," "Mathematical Lotto," and "Rocket" methods, which aim to engage students and enhance their learning experience. The article emphasizes the importance of reinforcing prior knowledge, particularly linear equations, to aid in solving logarithmic equations effectively. By using interactive and student-centered methods, the article seeks to simplify the learning process and improve students' understanding of logarithms and their applications.

Education is the most important and reliable way to acquire systematic knowledge. Education is characterized by two-way communication (learning and teaching), comprehensive development of personality and other features. Education is a unique cognitive process controlled by a teacher. The role of the teacher as a guide is reflected in the ability of students to fully master knowledge, skills and abilities that ensure the development of intellectual and creative abilities. Education is also the process of communication between the teacher and the students. He explains the content of the educational material to the students, gives questions and tasks, monitors their work,

identifies mistakes and shortcomings, corrects the mistakes made, and shows them how to work. Any education reflects the activities of the teacher and the student, i.e., the teacher's teaching and the student's learning-oriented activities, in other words, a direct, direct and relative relationship.

We know that "Solving logarithmic equations" is one of the most difficult topics for academic high school students. Before teaching this topic, it is necessary and necessary to repeat the previous topics. Because at the core of this topic lies linear equations with one unknown and methods of solving them. If the student is not sufficiently familiar with linear equations and how to solve them, these examples and problems will leave large gaps in solving. These topics play an important role in mathematics.

In order to repeat the logarithmic transformation of the previous topic, we use the "Find Mosini" method. If the equation includes a logarithm, such an equation is called a logarithmic equation.

| QUESTIONS | ANSWERS |
|---|-------------------|
| 1. $\log_3 10 \cdot \log_{10} 27$ | a) $\frac{25}{4}$ |
| 2. $(0,025)^{\lg 2} \cdot (0,04)^{\lg 2}$ | b) 3 |
| 3. $81^{0,5 \log_9 7}$ | c) 12 |
| 4. $\log_6 4 + \log_6 9 + \log_4 6 \cdot \log_{\sqrt{6}} 2 + 2$ | d) $\frac{1}{8}$ |
| 5. $\log_5 4 \cdot \log_6 5 \cdot \log_7 6 \cdot \log_8 7$ | e) $\frac{9}{7}$ |
| 6. $\log^2_{\sqrt[3]{5}} \sqrt{5} - \log_{\sqrt[3]{5}} 5\sqrt{5} + \log_{\sqrt{3}+1} (4 + 2\sqrt{3})$ | f) 5 |
| 7. $\log_b (a^2 b) = ? \quad \log_a b = 7$ | g) 7 |
| 8. $a = 3^b$ find the. If $b = \log_c 0,25 + 3 \log_u 4$, $u = 27$, $c = 1/9$ | h) $\frac{2}{3}$ |

Logarithmic equations are solved according to one of these theorems depending on their appearance.

Theorem 1: If $x = a^b$, then $a > 0, a \neq 1$ and $\log_a x = b$

For example:

$$a) \log_2 x = 4$$

$$b) \log_{\frac{1}{3}} x = -2$$

$$x = 2^4$$

$$x = \left(\frac{1}{3}\right)^{-2}$$

$$x = 16$$

$$x = 9$$

Theorem 2: If $f(x) = a^b$ $a > 0, a \neq 1$ and $\log_a f(x) = b$ then

For example:

$$\log_2 (3x - 1) = 4$$

$$3x - 1 = 2^4$$

$$3x = 17$$

$$x = 5\frac{2}{3}$$

Theorem 3: If $f(x) = g(x)$, $\log_a f(x) = \log_a g(x)$ and $a > 0, a \neq 1$ then

The roots found must satisfy the conditions $f(x) > 0$, $g(x) > 0$ and the given equation.

$$\begin{aligned} \lg(2x^2 - 4x + 12) &= \lg(x^2 + 3x) \\ 2x^2 - 4x + 12 &= x^2 + 3x \\ x^2 - 7x + 12 &= 0 \end{aligned}$$

$$x_1 = 3, x_2 = -4$$

2 la roots also satisfy the equation. Answer: 3 and 4

Theorem 4: If $g(x) = f(x)^b$ is, $\log_{f(x)} g(x) = b$ then is.

In this case, the roots found must satisfy the conditions and the given equation.

$$\begin{aligned} \log_x(5x - 4) &= 2 \\ x^2 &= 5x - 4 \\ x^2 - 5x + 4 &= 0 \end{aligned}$$

$x_1=1$ is an odd root, so there must be $x_1=4$. Answer: $x=4$

Note 1: Often, when solving logarithmic equations, it is recommended to first find the domain of the equation and solve the equation. But since the equation often has several roots, it is better to first solve the equation, put the found roots into the given equation and check and choose the answer.

Note 2: The given logarithmic equation is brought to any of the cases considered in these 4 theorems using formulas.

Some equations are solved only and only graphically.

I. Logarithmic equations reduced to a quadratic equation.

$A \cdot \log_a^2 x + B \cdot \log_a x + C = 0$ The equation is reduced to a quadratic equation with the help of notation, and the roots are found by solving it. $A \cdot y^2 + B \cdot y + C = 0$

$$\begin{aligned} 1) \log_a x &= y_1 & 2) \log_a x &= y_2 \\ x_1 &= a^{y_1} & x_2 &= a^{y_2} \end{aligned}$$

Example: $\log_3^2 x - 3 \cdot \log_3 x + 2 = 0$ solve the equation.

$$\log_3 x = y \text{ let's say } y^2 - 3y + 2 = 0 \quad y_1 = 1; \quad y_2 = 2$$

$$\begin{aligned} 1) \log_3 x &= 1 & 2) \log_3 x &= 2 \\ x_1 &= 3 & x_2 &= 9 \end{aligned}$$

This theorem is useful in solving many examples.

Theorem: $\log_a^2 x + p \cdot \log_a x + q = 0$ the product of the roots of the equation can be calculated as follows. $x_1 \cdot x_2 = a^{-p}$

II. Equations to be solved graphically.

Some logarithmic equations involve a linear or quadratic function in addition to logarithmic equations. For example:

$$a) \log_a x = kx + b \qquad b) \log_a x = kx^2 + bx + c$$

These equations cannot be solved analytically. These equations are solved graphically, and graphically, the roots of these equations can be found not exactly but approximately. But the number of roots of the equation can be found without error. For this, the left and right parts of the equation are considered as separate functions, and the graphs of these functions are described in 1 coordinate plane.

The more times its graphs intersect, the more roots the equation has.

When solving some equations, it is necessary to logarithmize both parts according to a base. According to what basis, it is necessary to determine the sequence of logarithms according to the equation.

III. Equations to be solved by logarithmization.

$$\begin{aligned}
 a) x^{\lg x - 1} &= 100 & \lg^2 x - \lg x - 2 &= 0 \\
 \lg x^{\lg x - 1} &= \lg 100 & 1) \lg x &= 2 \quad x = 100 \\
 (\lg x - 1) \cdot \lg x &= 2 & 2) \lg x &= -1 \quad x = \frac{1}{10}
 \end{aligned}$$

b) $2^{x^2} \cdot 3^x = 6$ if one root of the equation is 1, find the 2nd root.

Solution: Logarithmize both sides

$$\begin{aligned}
 \log_2(2^{x^2} \cdot 3^x) &= \log_2 6 & x_1 \cdot x_2 &= -\log_2 6 \\
 \log_2 2^{x^2} + \log_2 3^x - \log_2 6 &= 0 & 1 \cdot x_2 &= -\log_2 6 \\
 x^2 + x \cdot (\log_2 3) - \log_2 6 &= 0 & x_2 &= -\log_2 6
 \end{aligned}$$

Answer: $-\log_2 6$

Most logarithmic equations involve logarithms with different bases. In these equations, it is necessary to bring them to the same basis using "Transition formulas".

$$\log_a b = \frac{\log_c b}{\log_c a} \quad \log_a b = \frac{1}{\log_b a}$$

$$\begin{aligned}
 a) \log_{\sqrt{2}} x + \frac{2}{\log_x 2} &= 4 \\
 2 \log_2 x + 2 \log_2 x &= 4 & 4 \log_2 x &= 4 \\
 \log_2 x &= 1 & x &= 2
 \end{aligned}$$

Answer: 2

The topic can be reinforced to the students using different methods. For this topic, we will use the "Mathematical Lotto" method. For this method, we will need lotto stones. For the method, we will divide the group into 3 teams. We will write 4 numbers on 3 sheets.



Figure 1 Example of math lotto stones

To sheet -1:2,4,12,7

To sheet -2:5,6,9,3

To sheet -3:1,8,11,10

1 example of math lotto

$$1. \log_{2x+3} \frac{1}{4} + 2 = 0$$

$$2. \log_{2x+2} (2x^2 - 8x + 6) = 0$$

$$3. (2 - \log_6 x) \log_6 x = \frac{3}{4}$$

$$4. \lg x^2 + \lg \frac{2}{x} + \lg \frac{5}{x} = 4$$

$$5. \lg^2 x^2 - 3 \lg x - 1 = 0$$

$$6. \frac{1}{5 - \log_{\frac{1}{3}} x} + \frac{1}{1 + \log_{\frac{1}{3}} x} = 1$$

$$7. (\log_x \sqrt{5})^2 - \log_x (5\sqrt{5}) + 1,25 = 0$$

$$8. \frac{(\log_3 x)^2}{\log_3 (\frac{x}{27})} - \frac{6 - \log_3 x^5}{3 - \log_3 x} = 0$$

$$9. \log_5 x - \log_x 5 = \frac{3}{2}$$

$$10. (\log_2 x)^2 + 3 \log_{\frac{1}{2}} x + 2 = 0$$

$$11. 3 \log_8 (x + 1) = 8 + 3 \log_{x+1} 8$$

$$12. 5 \log_4 x + 3 \log_x 4 = 8$$

The next method is the rocket method

**Figure 2 “Rocket method”**

First, 3 rockets are drawn on the board. Each rocket is counted for each group. Under each rocket, 10 equations are written. Students divide into 3 groups and start solving the equations on the cards.



Figure 3 Sample cards for the "Rocket" method.

If students of any group answer the equations on the card in the rocket belonging to their group and if they answer correctly, the cards will be removed. The wrong card remains in the rocket and cannot fly. The first group to launch the rocket is considered the winner and 10 points are awarded. The group that failed to launch the rocket receives a penalty point.

Sample examples for the "Rocket Method".

1. $2\lg(x + \frac{1}{2}) - \lg(x - 1) = \lg(x + \frac{5}{2}) + \lg 2$
2. $\lg(x + \frac{4}{3}) - \lg(x - \frac{1}{3}) = \frac{1}{2}\lg(x + 6) - \frac{1}{2}\lg x$
3. $\log_5(x + 1) + \log_5(x + 5) = 1$
4. $\log_5(3x - 11) + \log_5(x - 27) = 3 + \log_5 8$
5. $\log_3(x^2 + 1) = \log_3 2 + \log_3(x + 8)$
6. $\log_9(2x^2 + 9x + 5) + \log_{\frac{1}{3}}(x + 3) = 0$
7. $\log_{49}(2x^2 + x - 5) + \log_{\frac{1}{7}}(x + 1) = 0$
8. $\log_4(x^2 - 4x + 2) - \log_4(x^2 - 6x + 5) = -\frac{1}{2}$
9. $\log_{\frac{1}{2}}(x^2 - 4x - 1) - \log_{\frac{1}{2}}(x^2 - 3x - 2) = -1$
10. $\log_2 \frac{x-2}{x-1} - 1 = \log_2 \frac{3x-7}{3x-1}$
11. $|\log_5 3 \cdot \log_3 x^4 - 5 \log_x x^2| = 2 \log_x 25$
12. $\sqrt{\log_2 |x| \cdot \log_2 (\frac{64}{|x|})} - 5 = \log_2 (\frac{x^2}{64})$

Summary. To summarize this article, it will help academic students learn logarithmic equations in a simple and easy way, quickly and easily. Through this article, students can study this topic independently [6-14].

Methodology

In teaching logarithmic equations to academic lyceum students, a variety of pedagogical methods were employed to ensure effective learning. The lesson plan focused on active student participation through interactive techniques such as the "Mosini Top," "Mathematical Lotto," and "Rocket" methods. These approaches aim to engage students by making the learning process more dynamic and enjoyable. The "Mosini Top" method was used to reinforce prior knowledge, particularly focusing on linear equations, which are crucial for solving logarithmic equations. The "Mathematical Lotto" method involved students working in teams to solve problems, while the "Rocket" method used a competitive game format to encourage quick thinking and problem-solving skills. The lesson aimed to cater to various learning styles and foster a deeper understanding of logarithmic equations through practical, hands-on activities.

Results and Discussion

The use of interactive methods such as the "Mosini Top," "Mathematical Lotto," and "Rocket" methods proved to be effective in engaging students and improving their understanding of logarithmic equations. Students actively participated, which helped reinforce the core concepts, particularly the transformation of logarithmic equations into solvable forms. The competitive nature of the activities also motivated students to solve problems faster and more efficiently.

However, some challenges were observed when solving more complex logarithmic equations graphically or through logarithmization. While students were able to grasp the basic concepts, additional practice was needed to master these more advanced techniques. Overall, the students' ability to apply theoretical knowledge to practical problems increased, demonstrating the success of using active learning strategies.

Conclusion

In conclusion, the application of modern pedagogical techniques such as "Mosini Top," "Mathematical Lotto," and the "Rocket" method significantly enhanced students' understanding of logarithmic equations. These methods not only made the learning process more engaging but also facilitated a deeper comprehension of complex mathematical concepts. By incorporating interactive and competitive learning strategies, students were able to better grasp the topic, solve problems efficiently, and apply their knowledge in real-world scenarios. Despite some challenges with more advanced techniques, the overall outcome demonstrated that these methods are highly effective for teaching logarithmic equations to academic lyceum students.

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